[1]W. A. Harrison, J. Phys. Chem. Solids 5, 44 (1958); see also J. M. Ziman, Advan. Phys. 13, 89 (1964).

[2]J. K. Mackenzie and E. H. Sondheimer, Phys. Rev. 77, 264 (1950).

[3]S. C. Hunter and F. R. N. Nabarro, Proc. Roy. Soc. (London) A220, 542 (1953).

[4]For a recent review and complete list of references, see F. R. N. Nabarro, *Theory of Crystal Dislocations* (Oxford U. P., London, 1967).

[5]The simple model used here is designed to simulate the scattering only due to the core of the dislocation. Inasmuch as the core of a screw dislocation also has a dilatation (see, for example, Ref. 4, p. 617), the model provides an estimate for the resistivity due to a screw dislocation also.

[6]These deviations turn out to be quite significant.

[7]A. B. Bhatia, Proc. Phys. Soc. (London) B62, 229 (1949).

[8]No confusion should arise from the use of the symbol P for the probabilities as well as for the Legendre polynomials.

[9]This form of solution is suggested by the fact that for the case where $C_L'' = C_0'' \delta_{L,0}$, the solution of Eqs. (33) is simply given by $f'_{L\eta} = \alpha(L) B_{1\eta} I(L, 1)$.

[10]Note the weighting factor in the integrand in (45) which is x^2, instead of the usual x^3 occurring in the expressions for the resistivity due to impurities, etc. [cf. Eq. (57)].

[11]J. M. Ziman, *Electrons and Phonons* (Oxford U. P., London, 1960).

[12]O. P. Gupta, Phys. Rev. 174, 668 (1968).

[13]J. G. Rider and C. T. B. Foxen, Phil. Mag. 13, 289 (1966).

[14]R. O. Simmons and R. W. Balluffi, Phys. Rev. 117, 62 (1960).

[15]R. M. J. Cotterill and M. Doyama, Phys. Rev. 145, 465 (1966); 150, 448 (1966).

[16]It is not entirely clear whether this is the only condition under which this procedure, i.e., averaging $P_1(\vec{k}, \vec{k}')$ over all orientations first and then solving the Boltzmann equation, will give exact result. For example, if the objects giving rise to anisotropic scattering $P_1(\vec{k}, \vec{k}')$ were of atomic dimensions (i.e., if their linear dimensions were all small compared with the electron mean free path), then one would automatically use this averaging process in preference to the one which averages over the resistivity.

[17]Z. S. Basinski, J. S. Dugdale, and A. Howie, Phil. Mag. 8, 1989 (1963).

[18]J. G. Rider and C. T. B. Foxon, Phil. Mag. 16, 1166 (1967).

[19]We may mention that a similar variation of $\langle \rho_D \rangle_{av}$ with temperature can occur even if $\rho_D(R)$ itself is independent of R, provided the dislocations are not quite randomly oriented. Consider the extreme case (perhaps, not likely to occur in practice) where the specimen is such that if it is divided into thin parallel slabs with their lengths along the direction of the current, then all the dislocations in a given slab are oriented parallel to one another while their orientations in different slabs are random. This corresponds to the slab resistances being in parallel, so that the average resistivity is $[\langle 1/\rho \rangle_{av}]^{-1}$, with ρ given by (54). For the case $(\rho_D)_3 = 0$, one then has the result that $\langle \rho_D \rangle_{av} = 0$ $\rho_i = 0$ and $\langle \rho_D \rangle_{av} = \frac{2}{3} (\rho_D)_{1,2}$ when $\rho_i \gg \rho_D$.

PHYSICAL REVIEW B VOLUME 1, NUMBER 12 15 JUNE 1970

Landau Diamagnetism from the Coherent States of an Electron in a Uniform Magnetic Field

Albert Feldman and Arnold H. Kahn
National Bureau of Standards, Washington, D. C. 20234
(Received 29 December 1969)

A complete set of coherent-state wave packets has been constructed for an electron in a uniform magnetic field. These states are nonspreading packets of minimum uncertainty that follow the classical motion. Use was made of the ladder operators that generate all the eigenstates of the Hamiltonian from any one energy eigenstate. The coherent states are the eigenstates of the two ladder operators that annihilate the zero-angular-momentum ground state. We have calculated the partition function, exploiting advantages of the coherent-state basis. The Landau diamagnetism and the de Haas–van Alphen oscillations are contained in the coherent-state framework.

I. INTRODUCTION

Coherent-state wave packets have received renewed attention since the recent article of Glauber.[1] Much of this attention is due to the recognition of their usefulness as a set of basis states for the calculation of observable physical quantities.[1,2] In addition, they have been of value in the theoretical problem of the quantum-mechanical definition of the phase of an oscillator.[3] The

coherent state is a wave packet whose probability distribution is invariant in time except for a displacive translation which obeys the classical equations of motion. Louisell[2] has summarized the physical and mathematical properties of the coherent states of the linear oscillator.

In this paper we have generated the complete set of coherent states of a charged particle in a magnetic field by use of step-ladder operators.[4] Some of these states were originally found by Darwin[5] in his examination of the classical action. In order to obtain all the states, we constructed ladder operators $X_\pm$ in addition to the ladder operators $\pi_\pm$ constructed by Johnson and Lippmann.[6] Spin was neglected throughout.

As an application, we have used the coherent states to calculate the magnetization and magnetic susceptibility of the free-electron gas and have thus rederived the Landau diamagnetism. The computation is aided by simplifications that occur when using the coherent states as a basis.

II. ENERGY EIGENVALUES AND EIGENSTATES

The energy eigenvalues and eigenvectors of a charged particle in a magnetic field were first found by Landau.[7] We shall review the problem in order to obtain the operators necessary for generating the coherent states. The notation of Kubo *et al.*[8] will be used. Two constants which characterize the problem are $\Omega = eH/\mu c$, the cyclotron frequency; and $l = (\hbar/\mu\Omega)^{1/2}$, the classical radius of the ground-state Landau orbit.

The Hamiltonian for a free electron in a magnetic field, neglecting spin, is

$$\mathcal{H} = \pi^2/2\mu \ , \tag{2.1}$$

where μ is the electron mass,

$$\vec{\pi} = \vec{p} + e\vec{A}/c \tag{2.2}$$

and $$\vec{H} = \nabla \times \vec{A} \ . \tag{2.3}$$

We choose the vector potential to be

$$\vec{A} = (-\tfrac{1}{2} Hy, \ \tfrac{1}{2} Hx, \ 0), \tag{2.4}$$

which corresponds to a uniform magnetic field parallel to the z direction. Thus, since the motion along the z direction is that of a free particle, it is necessary to solve only the two-dimensional problem in the xy plane. The appropriate Hamiltonian for the transverse motion is

$$\mathcal{H}_t = [(p_x - \tfrac{1}{2}\mu\Omega y)^2 + (p_y + \tfrac{1}{2}\mu\Omega x)^2]/2\mu \ . \tag{2.5}$$

If we introduce the operators[6]

$$\pi_\pm = p_x \pm i p_y \pm (i\hbar/2l^2)(x \pm i y) \tag{2.6}$$

which obey the commutation relation

$$[\pi_-, \ \pi_+] = 2\mu\hbar\Omega \ , \tag{2.7}$$

then $$\mathcal{H}_t = [(\pi_+\pi_-)/2\mu] + \tfrac{1}{2}\hbar\Omega \ . \tag{2.8}$$

This is mathematically equivalent to the linear oscillator Hamiltonian with the number operator being $\pi_+\pi_-/2\mu\hbar\Omega$. In addition, the angular momentum operator

$$L_z = x\, p_y - y\, p_x, \tag{2.9}$$

which has integer eigenvalues (in units of $\hbar$), commutes with $\mathcal{H}_t$. We select energy eigenstates that are simultaneously eigenstates of L_z. Labeling them $|Nm\rangle$, one obtains the eigenvalue equations

$$\mathcal{H}_t |N, m\rangle = (N + \tfrac{1}{2})\hbar\Omega |N, m\rangle \tag{2.10}$$

and $$L_z |N, m\rangle = m\hbar |N, m\rangle. \tag{2.11}$$

From the commutation relations of $\pi_\pm$ with $\mathcal{H}$ and L_z,

$$[\mathcal{H}, \ \pi_\pm] = \pm\hbar\Omega\pi_\pm \tag{2.12}$$

and $$[L_z, \ \pi_\pm] = \pm\hbar\,\pi_\pm, \tag{2.13}$$

we see the operators π_+ and π_- are the raising and lowering operators, respectively, for energy and angular momentum simultaneously. Thus

$$\pi_+ |N, m\rangle = (2\mu\hbar\Omega)^{1/2}(N+1)^{1/2} |N+1, \ m+1\rangle \tag{2.14}$$

and $$\pi_- |N, m\rangle = (2\mu\hbar\Omega)^{1/2}(N)^{1/2} |N-1, \ m-1\rangle \ . \tag{2.15}$$

Figure 1 demonstrates the properties of $\pi_\pm$. In the array of states $|N, m\rangle$ the $\pi_\pm$ operator can carry us along any one diagonal. It is evident that to generate all states from any particular state we must find an operator that will move us from one diagonal to another. An operator with this property can be constructed from the orbit center-coordinate operators[8]

$$X = x - (\pi_y/\mu\Omega) \tag{2.16}$$

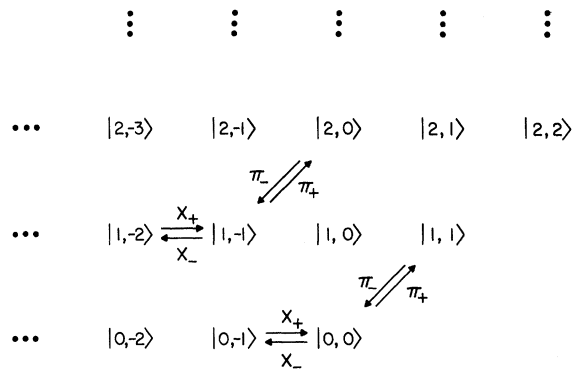

FIG. 1. Energy eigenstates $|N, m\rangle$ for the transverse motion of an electron in a magnetic field. The energy eigenvalue is $(N+\frac{1}{2})\hbar\Omega$ and the angular momentum eigenvalue is $m\hbar$. The stepping of the ladder operators operators $\pi_\pm$ and $X_\pm$ is indicated by arrows.

and $$Y = y + (\pi_x/\mu\Omega). \tag{2.17}$$

The operators

$$X_\pm = X \pm i\,Y \tag{2.18}$$

have the desired properties since they step only the angular momentum and not the energy, as shown by the commutation relations

$$[L_z, X_\pm] = \pm\hbar X_\pm \tag{2.19}$$

and $$[\mathcal{H}, X_\pm] = 0. \tag{2.20}$$

In addition, they commute with $\pi_\pm$. We can therefore show that

$$X_+|N, m\rangle = (\sqrt{2})\, l\, (N-m)^{1/2}\, |N, m+1\rangle$$

and $$X_-|N, m\rangle = \sqrt{2}\, l\, (N-m+1)^{1/2}\, |N, m-1\rangle.$$

It is clear that for a given energy $(N+\frac{1}{2})\hbar\Omega$, m can take on values N, $N-1$, . . . , 0, -1, . . . , $-\infty$, because the application of X_+ to the state $|N, N\rangle$ yields zero. All energy eigenstates can now be generated from the ground state $|0, 0\rangle$ by successive applications of π_+ and X_-; thus

$$|N, m\rangle = [(2\mu\hbar\Omega)^N\, (2l^2)^{N-m}\, N!\, (N-m)!\,]^{-1/2} \times X_-^{N-m}\, \pi_+^N |0, 0\rangle. \tag{2.21}$$

This is shown schematically in Fig. 1.

The ground-state zero-angular-momentum wave function, in coordinate representation, is obtained from the two conditions

$$\langle r|X_+|0,0\rangle = \left[l^2\left(\frac{\partial}{\partial x} + i\,\frac{\partial}{\partial y}\right) + \tfrac{1}{2}(x+iy)\right]\psi_{00} = 0,$$

$$\langle r|\pi_-|0,0\rangle = -i\hbar\left[\left(\frac{\partial}{\partial x} - i\,\frac{\partial}{\partial y}\right) + \frac{1}{2l^2}\,(x-iy)\right]\psi_{00} = 0\,.$$

The substitution $\rho_\pm = x \pm iy$ simplifies the solution. The result is

$$\psi_{00}(x, y) = [1/(\sqrt{2\pi})\, l\,]\, e^{-(x^2+y^2)/4l^2} = [1/(\sqrt{2\pi})\, l\,]\, e^{-\rho_+\rho_-/4l^2}$$

in which the state is normalized to unity over the xy plane.

III. COHERENT STATES

The coherent states of a harmonic oscillator have been discussed in detail by Glauber.[1] We follow his procedures for the problem of an electron in a magnetic field. We consider the transverse motion only. Let the coherent state $|\alpha, \xi\rangle$ be defined as the simultaneous eigenstate of the two commuting non-Hermitian operators which annihilate the ground state:

$$\pi_-|\alpha, \xi\rangle = (\hbar/i)\,(\alpha/l^2)|\alpha, \xi\rangle, \tag{3.1}$$

$$X_+|\alpha, \xi\rangle = \xi\,|\alpha, \xi\rangle. \tag{3.2}$$

The complex eigenvalues α and ξ have dimensions of length. Their physical significance and the coherence properties will follow.

We construct the coherent state by expanding in the complete set of energy eigenfunctions; thus

$$|\alpha, \xi\rangle = \Sigma\,|N, m\rangle\,\langle N, m|\alpha, \xi\rangle. \tag{3.3}$$

We can obtain the expansion coefficients $\langle N, m|\alpha, \xi\rangle$ in terms of the single coefficient $\langle 0, 0|\alpha, \xi\rangle$ by use of the Hermitian of Eq. (2.21) and Eqs. (3.1) and (3.2). Hence,

$$\langle N, m|\alpha, \xi\rangle = \frac{(\alpha/i)^N\, (\xi)^{N-m}}{(2l^2)^{N/2}(N!)^{1/2}(2l^2)^{(N-m)/2}[(N-m)!]^{1/2}} \times\langle 0,0|\alpha, \xi\rangle. \tag{3.4}$$

From the normalization condition

$$\langle\alpha, \xi|\alpha, \xi\rangle = 1\,, \tag{3.5}$$

we obtain the result

$$\langle 0,0|\alpha, \xi\rangle = \exp\left[-(|\alpha|^2 + |\xi|^2)/4l^2\right], \tag{3.6}$$

where an arbitrary phase factor has been set equal to unity.

By an additional use of Eq. (2.21), the coherent state can be put in the form

$$|\alpha, \xi\rangle = \exp\left[-\frac{1}{4l^2}\,(|\alpha|^2 + |\xi|^2) + \frac{\alpha\pi_+}{2i\hbar} + \frac{\xi X_-}{2l^2}\right]|0,0\rangle. \tag{3.7}$$

The time dependence of the coherent state is obtained by deriving the time dependence of X_+ and π_- in the Heisenberg picture. X_+ is a constant of the motion since it commutes with $\mathcal{H}$. On the other hand,

$$\pi_-(t) = e^{-i\Omega t}\,\pi_-(0), \tag{3.8}$$

which follows from the equation of motion

$$d\pi_-/dt = (1/i\hbar)\,[\pi_-, \mathcal{H}] = -i\Omega\pi_-. \tag{3.9}$$

Hence

$$\pi_-(t)|\alpha\xi\rangle = e^{-i\Omega t}\,\pi_-(0)|\alpha\xi\rangle = \alpha\, e^{-i\Omega t}\,|\alpha, \xi\rangle. \tag{3.10}$$

On transforming to the Schrödinger picture, the time-dependent coherent state becomes $|\alpha\, e^{-i\Omega t}, \xi\rangle$.

The Schrödinger representation of the coherent state $\psi_{\alpha\xi}(r)$ may be obtained in several ways. One method is to solve Eqs. (3.1) and (3.2) in the coordinate representation. Using $\rho_\pm = x \pm iy$, we obtain the differential equations

$$\left(\frac{\partial}{\partial\rho_+} + \frac{1}{4l^2}\,\rho_-\right)\,\psi_{\alpha\xi} = \frac{\alpha}{2l^2}\,\psi_{\alpha\xi}\,, \tag{3.11a}$$

$$\left(\frac{\partial}{\partial\rho_-} + \frac{1}{4l^2}\,\rho_+\right)\,\psi_{\alpha\xi} = \frac{\xi}{2l^2}\,\psi_{\alpha\xi}\,. \tag{3.11b}$$

Since the solution for $\alpha = \xi = 0$ is $e^{-\rho_+\rho_-/4l^2}$, we are

led to try simple translations of the variables ρ_+ and ρ_-. Thus we find

$$\psi_{\alpha\xi} \propto \exp\left[-(\rho_+ - 2\xi)(\rho_- - 2\alpha)/4l^2\right] e^{f(t)}, \quad (3.12)$$

where $f(t)$ is an undetermined function of time. We obtain $f(t)$ by requiring that $\psi_{\alpha\xi}$ satisfy the time-dependent Schrödinger equation with $\alpha(t)$ having the time dependence $e^{-i\Omega t}$. We find $f(t) = [\alpha(t)\xi/2l^2]$. Thus, the wave function, normalized to unity, is

$$\psi_{\alpha\xi} = \frac{1}{l(2\pi)^{1/2}} \exp\left[-\frac{1}{4l^2}(|\alpha|^2 + |\xi|^2) - \frac{1}{4l^2}(\rho_+ - 2\xi)(\rho_- - 2\alpha) + \frac{\alpha\xi}{2l^2}\right]. \quad (3.13)$$

Equation (3.13) can be obtained directly by finding the coordinate representation of Eq. (3.7). Use is made of the operator identities

$$\exp\left(x + c\frac{\partial}{\partial x}\right) = \exp(x)\exp\left(c\frac{\partial}{\partial x}\right)\exp(\tfrac{1}{2}c)$$

and $$\left[\exp\left(c\frac{\partial}{\partial x}\right)\right] f(x) = f(x+c).$$

The probability density in space is

$$|\psi_{\alpha\xi}|^2 = \frac{1}{2\pi l^2} \times \exp\left[-\left(\frac{1}{2l^2}\right)[(x - \alpha_1 - \xi_1)^2 + (y + \alpha_2 - \xi_2)^2]\right], \quad (3.14)$$

where $\alpha = \alpha_1 + i\alpha_2$, $\xi = \xi + i\xi_2$, and α_1, α_2, ξ_1, and ξ_2 are all real. Thus the coherent wave packet has the form of the ground state, but is displaced in space to a moving center. The mean coordinates are

$$\bar{x}(t) = \mathrm{Re}[\xi + \alpha(0)e^{-i\Omega t}], \quad \bar{y}(t) = \mathrm{Im}[\xi - \alpha(0)e^{-i\Omega t}]. \quad (3.15)$$

Thus the centroid of the packet follows a classical orbit of radius $|\alpha|$ around a center at (ξ_1, ξ_2). The shape of the packet is independent of time. The motion is depicted in Fig. 2. The use of the X_- operator enables us to place the center anywhere. These same techniques may be used to find the coherent states for crossed electric and magnetic fields, and for the harmonic oscillator in a magnetic field. These cases are discussed in Appendixes A and B.

The coherent states form a complete basis. Following Glauber,[1] we find that the closure relation may be expressed as

$$(1/4\pi^2 l^4)\int |\alpha, \xi\rangle\langle\alpha, \xi| d^2\alpha\, d^2\xi = 1, \quad (3.16)$$

where $d^2\alpha = d\alpha_1 d\alpha_2$; $d^2\xi = d\xi_1 d\xi_2$. As in the harmonic-oscillator case, these coherent states are states of minimum uncertainty, i.e., they satisfy uncertainty relations

$$\Delta x\, \Delta p_x = \Delta y\, \Delta p_y = \tfrac{1}{2}\hbar. \quad (3.17)$$

IV. DIAMAGNETIC SUSCEPTIBILITY

The coherent-state formulation permits us to use classical concepts for describing electron orbits, yet contains all quantum effects. We use this approach to calculate the partition function. The Landau diamagnetism is then obtained.

The partition function is given by

$$Z = \mathrm{Tr}\, e^{-\mathcal{H}/kT} = \sum\langle\alpha,\ \xi,\ k_z|\ e^{-\mathcal{H}/kT}|\alpha, \xi, k_z\rangle = \sum_{k_z} e^{-\hbar^2 k_z^2/2\mu kT}\int\frac{d^2\xi\, d^2\alpha}{4\pi^2 l^4}\ \langle\alpha, \xi| \times\left[\exp\left(-\frac{\pi_+\pi_-}{2\mu kT} - \tfrac{1}{2}\frac{\hbar\Omega}{kT}\right)\right]|\alpha, \xi\rangle. \quad (4.1)$$

The sum on k_z gives the usual partition function for one-dimensional free motion. We evaluate Z for a cylindrical body of length L, radius R, oriented along the magnetic field. Thus we have

$$Z = Z_\parallel Z_\perp, \quad (4.2)$$

where $$Z_\parallel = (L/h)(2\pi\mu kT)^{1/2}. \quad (4.3)$$

The transverse part may be simplified through the properties of the coherent states. We use the boson-operator identity[2]

$$e^{xa^\dagger a} = \sum_{n=0}^{\infty}\frac{(e^x - 1)^n}{n!}\, a^{\dagger n} a^n, \quad (4.4)$$

where a and $a^\dagger$ satisfy $[a, a^\dagger] = 1$, to evaluate Eq. (4.1). Thus the factor

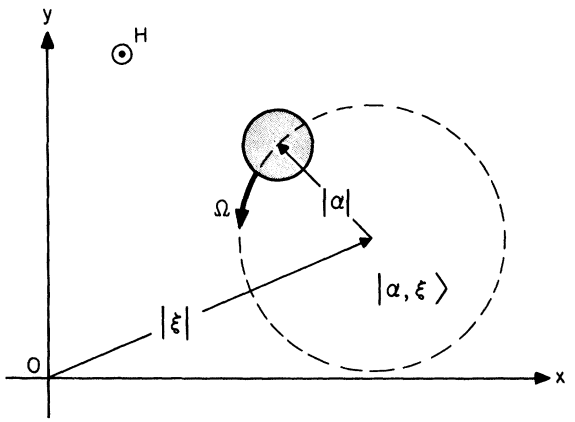

FIG. 2. Motion of the coherent state $|\alpha, \xi\rangle$. The shaded region represents the Gaussian packet which follows the classical motion of cyclotron frequency Ω.

$$\left\langle \alpha\xi \left| \left[\exp\left(-\frac{\pi_+\pi_-}{2\mu kT} \right) \right] \right| \alpha\xi \right\rangle = \exp\left(\frac{|\alpha|^2}{2l^2} \left(e^{-\hbar\Omega/kT} - 1 \right) \right) \tag{4.5}$$

and $$Z_\perp = e^{-\hbar\Omega/2kT} \times \int \frac{d^2\xi\, d^2\alpha}{4\pi^2 l^4} \exp\left(-\frac{|\alpha|^2}{2l^2} \left(1 - e^{-\hbar\Omega/kT} \right) \right) . \tag{4.6}$$

To perform the integration over ξ and α, we exclude all coherent states with $|\alpha+\xi| > R$, i.e., we sum over all orbits lying within the cylinder. The limitation on the terms to be included in the sum over coherent states is equivalent to the cutoff of energy eigenstates used by Landau.[7] However, since $R \gg l$, for our application, and the exponential falls off rapidly with $|\alpha|^2$, we can safely extend the α integration to infinity. The results are

$$Z_\perp = \frac{e^{-\hbar\Omega/2kT}}{4\pi^2 l^4} \int_0^R 2\pi|\xi|d|\xi| \int_0^\infty 2\pi|\alpha|d|\alpha| \times \exp\left(-\frac{|\alpha|^2}{2l^2} \left(1 - e^{-\hbar\Omega/kT} \right) \right) \tag{4.7}$$

and $$Z = V \frac{(2\pi\mu kT)^{1/2}}{h} \frac{\mu\Omega}{4\pi\hbar} \frac{1}{\sinh(\hbar\Omega/2kT)} . \tag{4.8}$$

The magnetization M is obtained from the free energy F by

$$F = -nkT \ln Z \tag{4.9}$$

and $$M = -\frac{\partial F}{\partial H} = \frac{ne\hbar}{\mu c} \left(\frac{kT}{\hbar\Omega} - \frac{1}{2} \coth \frac{\hbar\Omega}{2kT} \right) . \tag{4.10}$$

The susceptibility χ, per electron, in the high-temperature limit is the correct Landau diamagnetism

$$\chi = -\frac{1}{n} \frac{\partial M}{\partial H} = -\frac{1}{3} \left(\frac{e\hbar}{2\mu c} \right)^2 \frac{1}{kT} . \tag{4.11}$$

Once the partition function is known for the nondegenerate case, the free energy for the free-electron gas with Fermi statistics may be found directly, as shown by Wilson.[9] This is because the density of states is given by the inverse Laplace transform of the partition function.[10] Hence all quantum effects, including the de Haas–van Alphen effect, are contained in the coherent-state treatment.

In Appendix B we obtain Z for an oscillator in a magnetic field.[11] In this case there is no need for a cutoff at radius R. In the limit of small spring constant we obtain the same M and χ as obtained above.

In this section the coherent state enabled us to use classical concepts of orbit center and radius. The quantum effects arose through the noncommutability of π_+ and π_-. This led to a distribution of orbit radii of the same form that Bloch[12] first derived for the amplitude distribution of a harmonic oscillator in thermal equilibrium.

APPENDIX A: CROSSED ELECTRIC AND MAGNETIC FIELDS

In crossed electric and magnetic fields the Hamiltonian is augmented by a term

$$V = e\vec{E}\cdot\vec{r} = e(E_x x + E_y y) = \tfrac{1}{2}e\,(E_+\rho_- + E_-\rho_+),$$

where $E_\pm = E_x \pm iE_y$. The Heisenberg equations of motion for the operators π_- and X_+ now become

$$d\pi_-/dt = -(i/\hbar)[\pi_-,\ \mathcal{H}_t + V] = -i\Omega\pi_- - eE_-$$

and $$dX_+/dt = -(i/\hbar)[X_+,\ V] = -ieE_+/\mu\Omega .$$

The solutions are

$$\pi_-(t) = e^{-i\Omega t}\pi_-(0) - eE_-/i\Omega ,$$

$$X_+(t) = X_+(0) - (ieE_+/\mu\Omega)t.$$

Thus the coherent state $|\alpha_0,\ \xi_0\rangle$ at $t = 0$ will have evolved at time t to the state

$$\left| \alpha_0 e^{-i\Omega t} - \frac{eE_-}{\mu\Omega^2},\ \xi_0 - \frac{ieE_+}{\mu\Omega} t \right\rangle .$$

The location of the centroid at time t is given by

$$\bar{x}(t) = \mathrm{Re}\left(\xi_0 - \frac{ieE_+}{\mu\Omega} t + \alpha_0 e^{-i\Omega t} - \frac{eE_-}{\mu\Omega^2} \right) ,$$

$$\bar{y}(t) = \mathrm{Im}\left(\xi_0 - \frac{ieE_+}{\mu\Omega} t - \alpha_0 e^{-i\Omega t} - \frac{eE_-}{\mu\Omega^2} \right) .$$

This motion is the cycloidal path followed by a classical electron in crossed fields. The coherent state has the same charge distribution as before, but follows this new path.

APPENDIX B: ELECTRON IN A UNIFORM MAGNETIC FIELD AND A HARMONIC POTENTIAL

The energy eigenvalues and eigenfunctions for this problem were originally found by Fock.[11] The Hamiltonian $\mathcal{H}_t$ is

$$\mathcal{H}_t = [(p_x - \tfrac{1}{2}\mu\Omega y)^2 + (p_y + \tfrac{1}{2}\mu\Omega x)^2]/2\mu + \tfrac{1}{2}\mu\omega_0^2(x^2+y^2).$$

If we make the substitution

$$\Omega' = (\Omega^2 + 4\omega_0^2)^{1/2},$$

we then obtain

$$\mathcal{H}_t = \frac{1}{2}\left[\frac{\pi_+\pi_-}{2\mu} \left(1 + \frac{\Omega}{\Omega'} \right) + \frac{1}{2}\mu\Omega'^2 X_- X_+ \left(1 - \frac{\Omega}{\Omega'} \right) + \hbar\Omega' \right] , \tag{B1}$$

where Ω' is substituted for Ω in the definitions of l, $X_\pm$, and $\pi_\pm$. The energy eigenstates of Eq.

(B1) are the same as before, but the eigenvalues are not $\{(N+\frac{1}{2})\hbar\Omega' - m\hbar[\frac{1}{2}(\Omega'-\Omega)]\}$. The coherent states are also the same as before except that both α and ξ will depend on time. Thus

$$\alpha(t) = \alpha(0)\exp\{-i[\tfrac{1}{2}(\Omega'+\Omega)]t\},$$
$$\xi(t) = \xi(0)\exp\{-i[\tfrac{1}{2}(\Omega'-\Omega)]t\}.$$

The wave packet will therefore follow the classical motion.

To calculate $Z_\perp$ in this case, one may take all the limits of integration from 0 to ∞. The partition function is then

$$Z_\perp = \left(4\sinh\frac{\hbar(\Omega'-\Omega)}{4kT}\sinh\frac{\hbar(\Omega'+\Omega)}{4kT}\right)^{-1}.$$

For $\omega_0 \ll \Omega$, $Z_\perp$ becomes

$$Z_\perp = \left(\frac{2\hbar\omega_0^2}{kT\,\Omega}\sinh\frac{\hbar\Omega}{2kT}\right)^{-1}.$$

Except for a proportionality constant, this equation is the same as Eq. (4.8) and therefore leads to the same magnetization and susceptibility.

[1]R. J. Glauber, Phys. Rev. 131, 2766 (1963).

[2]W. Louisell, *Radiation and Noise in Quantum Electronics* (McGraw-Hill, New York, 1964).

[3]P. Carruthers and M. N. Nieto, Rev. Mod. Phys. 40, 411 (1968).

[4]After the completion of this manuscript, it came to the attention of the authors that coherent states of the same type have been constructed by I. A. Malkin and V. I. Man'ko, Zh. Eksperim. i Teor. Fiz. 55, 1014 (1968) [Soviet Phys. JETP 28, 527 (1969)].

[5]G. C. Darwin, Proc. Roy. Soc. (London) 117, 258 (1928).

[6]M. H. Johnson and B. A. Lippmann, Phys. Rev. 76, 828 (1949).

[7]L. D. Landau, Z. Physik 64, 629 (1930).

[8]R. Kubo, S. J. Miyake, and N. Hashitsume, Solid State Phys. 17, 269 (1965).

[9]A. H. Wilson, *The Theory of Metals* (Cambridge U. P., Cambridge, England, 1953), pp. 160–172.

[10]C. Kittel, *Elementary Statistical Physics* (Wiley, New York, 1958), p. 56.

[11]V. Fock, Z. Physik 47, 446 (1928).

[12]F. Bloch, Z. Physik 74, 295 (1932).

PHYSICAL REVIEW B VOLUME 1, NUMBER 12 15 JUNE 1970

Spin-Polarized Energy Bands in Eu Chalcogenides by the Augmented-Plane-Wave Method

S. J. Cho

Division of Pure Physics, National Research Council of Canada, Ottawa, Canada

(Received 22 October 1969)

Spin-polarized energy bands in the Eu chalcogenides have been obtained by the augmented-plane-wave (APW) method. The $4f$ band positions are extremely sensitive to the exchange potential used. A reduced exchange parameter of $\frac{3}{4}$ for the magnetic Eu^{2+} ions has produced proper energy gaps and relative f band positions for EuO, EuS, and EuSe. We have obtained the $f(\uparrow)$ bandwidth as about 0.5 eV and the up- and down-spin f band separation as about 6 eV. We have also obtained anion p bandwidths of about 2 eV which are almost constant for the Eu chalcogenides. The calculated density of states agrees qualitatively with photoemission data, except for the experimental density of states of the $4f(\uparrow)$ band which has a large bandwidth of about 1.5 eV. The probable causes of this discrepancy are multiple scattering of the $4f$ electrons with phonons and electrons, or recombinations with Eu^{3+} ions. The observed absorption-edge red shifts are due to the spin-polarized exchange splitting ΔE_{ex} of the lowest conduction band X_3. Estimated ΔE_{ex} values are 0.4–0.5 eV for Gd metal and Eu chalcogenides. The $f(\uparrow)$ bands in Eu and Gd metals are expected to be located within 3 eV below the Fermi level. The high-energy reflectivity data, effective masses, possible conduction mechanism, and APW charge analysis have been discussed.

I. INTRODUCTION

Recently an increasing number of authors have studied the Eu chalcogenides, both experimentally and theoretically. Eu chalcogenides have a simple NaCl structure and are magnetic semiconductors. Despite extensive studies of the rare-earth mate-